lying the theoretical development are certainly sound in general, and since the previous comparison of the dissociation case theory with Bernfeld's data suggested strong support for that situation, we may at least have some confidence in the applicability of Equation (10) to real systems.

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Effects of Surface Tension and Gravity Forces in Determining The Stability of Isothermal Fiber Spinning

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Under certain conditions during the melt spinning of synthetic fibers, an instability known as draw resonance can occur (Miller, 1963). The instability can be seen as a periodic oscillation in filament thickness at a fixed point in the drawdown zone. Such a thickness oscillation increases in amplitude as the take-up speed is increased. The onset of resonance occurs at a critical value of the draw ratio (ratio of take-up speed to speed at the die exit) E_{crit}.

E_{crit}, in turn, can depend upon rheological properties of the fluid (Pearson and Shah, 1974; Fisher and Denn, 1976; Weinberger et al., 1976) and upon spinning conditions (Shah and Pearson, 1972). For example, E_{crit} for a Newtonian fluid is predicted theoretically to depend upon inertia, gravity, and surface tension forces, as well as upon thread-line cooling. However, for a Newtonian fluid spun isothermally under conditions where viscous forces dominate, the critical draw ratio E_{crit} is predicted to be a constant, 20.21 (Kase, 1966; Pearson and Matovich, 1969). This value has been recently corroborated by experimental tests with a Newtonian silicone oil (Donnelly and Weinberger, 1975).

Shah and Pearson (1972), using a linearized perturbation analysis, obtained quantitative predictions of the effects of inertia, surface tension, and gravity forces upon E_{crit}. Shah and Pearson, hereafter S-P, obtained numerical solutions to the steady state and perturbation equations of motion and continuity for the boundary conditions of constant take-up and die speeds. These solutions enabled them to generate the three functions $E_{crit}(Re)$, $E_{crit}(Re/Fr)$, $E_{crit}(Re/We)$, corresponding to the three separate cases of nonzero Re, Re/Fr, or Re/We. Here, the dimensionless groups representing the ratios of inertial, gravity, and surface tension forces to viscous forces, respectively, are

$$Re = \frac{\rho L \bar{v}_o}{3\eta}, \qquad \frac{Re}{Fr} = \frac{\rho g L^2}{3\eta v_o}, \qquad \frac{Re}{We} = \frac{\sigma L}{6\eta a_o \bar{v}_o} \tag{1}$$

S-P found that E_{crit} increased with increasing Re and Re/Fr and decreased with increasing Re/We. In the limit of large Re/Fr, one approaches the conditions of a freely falling stream, and the increased stability for this case is perhaps expected. Similarly, large Re/We corresponds to large (relatively) surface tension forces, and the decreased stability for this case is likewise perhaps expected. These predictions are important since they suggest means whereby draw resonance can be avoided by suitable modification of the spinning conditions. Accordingly, it becomes important to test these predictions; the present work describes the results of such tests of the effects of gravity and surface tension forces upon E_{crit}.

EXPERIMENTAL

The experimental apparatus and procedure were identical to those used earlier by Donnelly and Weinberger (1975). Briefly, the apparatus consisted of a vertically mounted syringe pump and a variable speed takeup roll. The fluid used was a Newtonian silicone oil, $\sigma = 22.4 \times 10^{-3}$ N/m, $\eta = 100$ N s/m^2. The take-up speed was increased until thread-line oscillation could be observed, and photographs of the thread line at moments of maximum and minimum thickness (at a fixed location 40% of the distance from the die to the take-up roll) were taken at stepwise increments of the take-up speed (or E). Plots of the resulting diameter ratio DR as a function of E were extrapolated to the line $DR = 1$; the intercept thus defines E_{crit}. Temperatures were maintained at 25 ± 1°C.

The apparatus could not be readily modified to permit investigation of inertial effects (Re); however, the effects of gravity (Re/Fr) and surface tension (Re/We) could be evaluated. Re/Fr was varied from 10 to 70, while Re/We ranged from 0.2 to 0.81. Note that neither gravity nor surface tension was varied directly but rather the ratios of gravity and surface tension forces to viscous forces.

RESULTS AND DISCUSSION

The experimental conditions and results are listed in Table 1 and the results are summarized in Figure 1, where E_{crit} is plotted vs. Re/Fr, with Re/We a parameter. In all cases, $Re < 4 \times 10^{-3}$, which is well below the Re of 3×10^{-2} where inertial effects, as predicted by Shah and Pearson, should become important. Two curves, one for

TABLE 1. EXPERIMENTAL CONDITIONS AND RESULTS

Run No.	Q (mm^3/s)	d_o (mm)	L (mm)	Re/Fr	Re/We	E_{crit}
D-W 1*	30.8	1.4	20	2.6	0.11	17.2
D-W 4*	30.8	1.4	40	10	0.21	17.2
1	44.5	1.2	78	20	0.25	26.0
2	44.5	1.2	96	30	0.30	37.1
3	44.5	1.2	109	39	0.35	46.2
4	20.4	0.93	73	23	0.39	26.9
5	30.7	1.2	85	34	0.39	43.0
6	44.5	1.2	124	50	0.39	52.0
7	6.95	0.75	64	33	0.81	17.8
8	10.1	1.2	58	48	0.81	31.9
9	14.1	0.75	129	69	0.81	46.9

* From Donnelly and Weinberger (1975).

TABLE 2. VARIATION OF SEVERITY OF RESONANCE WITH Re/Fr AND N_{De} (LOW Re/We)

Run No.	Re/Fr	Severity dDR/dE	N_{De}
D-W 1*	2.6	0.50	0.036
D-W 4*	10	0.04	0.009
1	20	0.04	0.009
2	30	0.05	0.007
3	39	0.04	0.006
5	34	0.04	0.006
6	50	0.03	0.006

* From Donnelly and Weinberger (1975).

$Re/We < 0.39$ and the other for $Re/We = 0.81$, were obtained. For each curve, E_{crit} increases with Re/Fr, in qualitative agreement with the theoretical predictions obtained by S-P. However, the S-P analysis indicated that E_{crit} should begin to increase when Re/Fr reaches a value of 0.5, whereas the experimental results show that such an increase does not occur until Re/Fr reaches roughly 10. Also, E_{crit} is predicted to increase with Re/Fr much more rapidly than was observed. Conceivably, these differences can be attributed to possible interaction effects among Re, Re/Fr, and Re/We. Recall that the function $E_{crit}(Re/Fr)$ was obtained for the case of Re and Re/We equal to zero. Examining the experimental results closely, however, there appears to be no influence of Re/We for Re/We in the range 0.2 to 0.39; this suggests that interactional effects are minimal. In any case, the predicted stabilizing effect of gravity was definitely observed, although the effect becomes significant at a larger Re/Fr than indicated by Shah and Pearson.

For the case of surface tension effects, the quantitative agreement between the S-P predictions and experimental results is much better. According to S-P, increasing Re/We from 0.30 to 0.81 should decrease E_{crit} by 46%, assuming no interactional effects with Re/Fr. From Figure 1, we observe roughly a 50% decrease in E_{crit} for the same change in Re/We. The destabilizing effect of surface tension persists for a range of values of Re/Fr. Since the effect is roughly constant, as indicated by the approximately parallel lines of Figure 1, interaction effects do not appear significant.

Donnelly and Weinberger (1975), (D-W), found that the severity of resonance, expressed as dDR/dE, was greater at higher tensile stresses. Since higher tensile stress corresponds to the case where viscous stresses are relatively large, or where the ratio of Re/Fr is small, it may be more appropriate to associate dDR/dE with Re/Fr. The results in Table 2 indicate that dDR/dE decreases as Re/Fr increases, in qualitative agreement with the earlier speculation of D-W. Fisher and Denn (1975), by completing a nonlinear stability analysis, obtained a value of

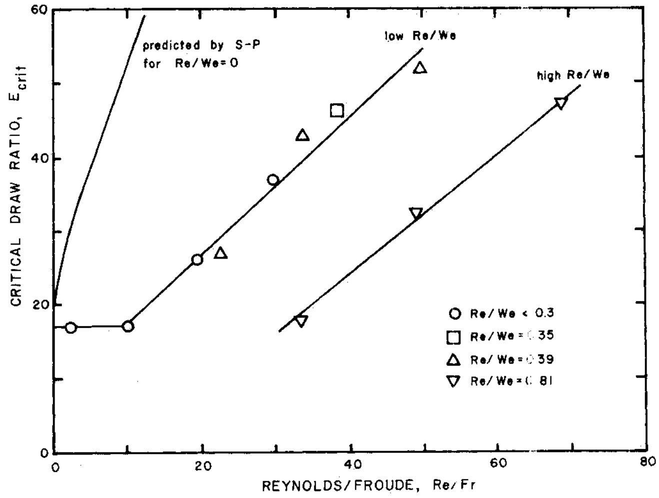

Fig. 1. Observed dependence of E_{crit} upon Re/Fr and Re/We.

0.34 for dDR/dE for the case of negligible Re, Re/Fr, and Re/We. This numerical analysis would have to be extended to include finite Re/Fr in order to check the experimentally observed decrease in dDR/dE with increasing Re/Fr.

Fisher and Denn also examined the influence of elastic response by computing dDR/dE for a Maxwell type of fluid model. Defining a Deborah number

$$N_{De} = \frac{\lambda v_o}{L} \tag{2}$$

they found that dDR/dE was roughly 0.15 for $N_{De} = 0.006$ (and E_{crit} was unchanged); therefore, the predicted effect of elastic response is a diminution of severity of resonance. Since the silicone oil used, a polydimethylsiloxane, is slightly viscoelastic, it becomes important to examine our results in terms of possible elastic effects. Previous stress-relaxation tests (Weinberger, 1970) and shear viscosity/shear rate curves indicate that $\lambda \leq 0.06$ s for this fluid. With $\lambda = 0.06$ used, computed Deborah numbers ranged from 0.006 to 0.036. These values, along with severity of resonance dDR/dE, are listed in Table 2. The results suggest that dDR/dE decreases with decreasing N_{De}, in apparent contradiction with the predicted results of Fisher and Denn. A plausible explanation of the discrepancy is that elastic response was indeed insignificant in our experimental tests and that the observed dDR/dE variation can be attributed to variation in Re/Fr. As noted earlier, additional numerical computation would be required to check this hypothesis.

In summary, the experimental findings confirm qualitatively the theoretical predictions of a stabilizing effect of gravity, although such an effect becomes significant at a larger Re/Fr than predicted theoretically. Quantitative agreement between experiment and theory was obtained for the destabilizing effect of surface tension (Re/We).

NOTATION

- a_o = die radius
- d_o = die diameter
- DR = filament diameter ratio
- E = draw ratio, $v_{take\text{-}up}/v_o$
- E_{crit} = critical draw ratio
- Fr = Froude number, Equation (1)
- L = length of drawdown zone
- N_{De} = Deborah number, Equation (2)
- Q = volumetric flow rate
- Re = Reynolds number, Equation (1)
- v = filament velocity

$\bar{v}_o$ = mean die velocity
We = Weber number, Equation (1)

Greek Letters

η = viscosity
λ = characteristic time of the fluid
σ = surface tension

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Mass Transfer From Swarms of Bubbles or Drops With Chemical Reactions in Continuous Phase

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Mass transfer with chemical reactions between the continuous phase and a single bubble or drop moving at a low or high Reynolds number has been investigated previously by Ruckenstein, Dang, and Gill (1971, 1973). They considered the rate determining step of the transfer processes was in the continuous phase. Gal-Or and Yaron (1973) studied physical mass transfer between ensembles of drops or bubbles and the continuous phase.

In many practical systems of interest, mass transfer with chemical reactions usually takes place between swarms of bubbles and continuous phase. Therefore, study and understanding of those systems are of extreme importance.

The objective of the first part of the present paper is to analyze the problem of mass transfer from swarms of bubbles with chemical reactions. The second part of the paper is to investigate experimentally a bubble mass transfer system.

When resistance for mass transfer is considered to be in the continuous phase, one can write the following unsteady convective diffusion equation with first-order chemical reaction for swarms of bubbles moving at low Reynolds number:

$$\frac{\partial c}{\partial t} + V_r \frac{\partial c}{\partial r} + \frac{V_\theta}{r}\frac{\partial c}{\partial \theta} = D\frac{\partial^2 c}{\partial r^2} - kc \tag{1}$$

The initial and boundary conditions are

$$c(o, r, \theta) = c_o \tag{2}$$

$$c(t, \infty, \theta) = c_o e^{-kt} \tag{3}$$

$$c(t, a, \theta) = c^* \tag{4}$$

and the velocity profile for swarms of bubbles has been established by Gal-Or and Yaron (1973) as

$$V_r = \frac{3}{2}\frac{U_s}{A}\left[\left(\frac{r}{a}\right)^2 \Phi^{5/3} - B + A\left(\frac{r}{a}\right)^{-1} - \left(\frac{r}{a}\right)^{-3}\right]\cos\theta \tag{5}$$

$$V_\theta = \frac{3}{2}\frac{U_s}{A}\left[\frac{-1}{2}\left(\frac{r}{a}\right)^{-3} - \frac{A}{2}\left(\frac{r}{a}\right)^{-1} + B - 2\left(\frac{r}{a}\right)^2 \Phi^{5/3}\right]\sin\theta \tag{6}$$

$$1 \leq \frac{r}{a} \leq \Phi^{-1/3}$$

where

$$A = 3 + 2\beta + 2\Phi^{5/3}(1-\beta),$$

$$B = 2 + 2\beta + \Phi^{5/3}(3 - 2\beta), \quad \beta = \frac{\mu_c}{\mu_d + \gamma} \tag{7}$$

The first step toward solving this set of equations is to introduce some dimensionless variables as

$$\omega = \frac{c}{c^*}, \quad Y = \frac{r-a}{a}, \quad \tau = \frac{tD}{a^2}, \quad R = \frac{ka^2}{D},$$